Around Orthogonal Calculus

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About Orthogonal Calculus:
Let J denote the enriched category whose objects are finite-dimensional, real inner product spaces and whose morphism spaces are the spaces of linear, isometric embeddings. Several interesting families of objects in Algebraic Topology can be described as continuous (in the enriched sense) functors from J to $\mathcal{T}$, where $\mathcal{T} \in \{\text{spaces, pointed spaces, spectra}\}$. Examples include

- $V \mapsto BO(V)$
- $V \mapsto BTOP(V)$
- $V \mapsto BG(V)$ [1]
- $V \mapsto B\mathrm{Diff}(V)$
- $V \mapsto \Sigma^\infty_+ \Omega J(W, W \oplus V)$

Orthogonal Calculus, a homotopy-theoretical tool first introduced by Michael Weiss, relates to $\mathcal{E} := Fun(J, \mathcal{T})$ in a similar way as usual calculus relates to $C^\infty(\mathbb{R})$. In particular, it defines what it means for $F \in \mathcal{E}$ to be polynomial of degree n and constructs for any F a series of better and better approximations (analagous to the Taylor series).

About this talk:
I will define the basic notions and state the most important properties of Orthogonal Calculus (and state or prove some of them). I will also explain how this theory can be expressed nicely using the framework of ∞-categories (which was not yet available when it was conceived), and explain one particular consequence of this new setup. Finally, I will try to explain a direction in which I hope to improve this theory further.

[1] $G(V)$ denotes the topological monoid of homotopy automorphisms of $S(V)$.